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IDENTIFICATION OF ERRORS-IN-VARIABLES MODELS WITH FAURRE TYPE REALIZATION ALGORITHMS

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Abstract—Identification of multiple input multiple output discrete time linear dynamic systems operating in open or closed loop are considered in the time invariant case. Two methods have been used for such a purpose: the recursive prediction error method on the input–output data and the successive Gram–Schmidt orthogonalization of the spectral density function of the joint input–output variable.

1. INTRODUCTION

Identification and identifiability of multivariable systems are discussed by many authors e.g. Refs [1–13]. Most of these results are valid for the case when there are no errors in the variables or when the noisy system is identified. We discuss the problem of computing the model of the true system from the covariance functions of the noisy data when the covariances of the mutually uncorrelated white measurement noises are given and these noises are uncorrelated with input–output signals. In this case the noise-free covariances are computable from the noisy covariances of the input–output data. A modified version of Faurre's realization algorithm [1] was given by the authors [2]. This modified version computes the Markov parameters of the transfer function in a direct way when the spectral density function of a stationary time-series is factored. These type of algorithms seem to be suitable devices for modelling errors-in-variables models (EIV models) proposed by Kalman [3]. This problem is investigated in this paper. The essence of the realization approach is: (i) compose a Hankel matrix based on the covariances of some appropriate random variables; (ii) factor the elements of this Hankel matrix with the Ho–Kalman algorithm [4]; (iii) using some Riccati equations we obtain the required parameter and covariance matrices of the state space description of the considered transfer function. Desai and Pal [5] presented a canonical realization algorithm where the covariance matrix of the state vector is diagonal and contains the canonical correlations discussed below. This realization is called balanced and a procedure was given for the approximation of the dimension of the state vector based on the mutual information between the past and the future of the considered process.

According to Picci and Pinzoni [6] and Deistler [7] the EIV model of the joint input–output variable can not be uniquely determined from the second moments of the observations. A typical area of the application is the field of econometrics where it is very often not clear what variables are "endogenous" and what are "exogenous". In this situation the causality relation among the variables is not obvious *a priori*. Against this fact the causality relation is necessary to identify industrial processes operating under feedback.

Recall from Picci–Pinzoni [6] that the weakly stationary input process $u(t) \in R^k$ causes the weakly stationary output process $y(t) \in R^m$, $t = 1, 2, \ldots,$ if and only if

$$y(t) = T_d(z)u(t) + s(t) = T_d u(t) + T_s(z)e_s(t), \tag{1}$$

where $T_d(z)$ is an $m \times k$ causal matrix transfer function and $s(t)$ is a stationary process completely independent of $u(t)$ i.e. $Eu(t)s^T(r) = 0$, for all $t, r = 1, 2, \ldots,$ and where E denotes expectation, z is forward shift operator and T stands for transpose.

The process $s(t)$ can be expressed by its innovation representation where $T_s(z)$ is a minimum phase transfer function and normalized so as to make $T_s(\infty) = I$. It is assumed that the $T_s(z)$

transfer function is rational when the above innovation representation has the form of the well-known Kalman filter as well.

The input process has the innovation representation $T_u(z)e_u(t)$, where $T_u(z)$ is a minimum phase rational transfer function and $T_u(\infty) = I$. The $e_s(t)$ and $e_u(t)$ are mutually independent full rank white noise processes.

These noise processes are the one step ahead prediction errors and they contain the measurement noises. In this way the identification of the $T_u(z)$, $T_d(z)$ and $T_s(z)$ transfer functions gives us a noisy system model and the computation is possible with Faurre type realization algorithms as well as recursive prediction error method of Ljung *et al.* [8].

A causal EIV model of the open loop system is computable with Faurre type realization algorithms. This follows from the properties of these algorithms as it is discussed in Section 2. In Section 3 some comments on the identifiability of closed loop systems are given. With Faurre type realization algorithms on the joint variable only a noncausal EIV model can be identified.

The causal noisy model in the closed loop case is a stochastic feedback scheme

$$\begin{aligned} y(t) &= T_d(z)u(t) + T_s(z)e_s(t), \\ u(t) &= T_c(z)y(t) + T_u(z)e_u(t), \end{aligned} \tag{2a}$$

where at most the $e_s(t)$ and $e_u(t)$ innovations can be assumed uncorrelated to the past histories of $y(t)$ and $u(t)$ and with each other. This means that there is delay in the system, i.e. $T_d(\infty) = 0$ and imposes *a priori* a causality relation on the data. Identification and identifiability of such feedback schemes are discussed in Refs [8–11].

In the case of EIV model equation (2a) has the form of (see Fig. 1.)

$$\begin{aligned} y(t) - e_0(t) &= T_d^+(z)[u(t) - e_I(t)] + T_s^+(z)e_s^+(t), \\ u(t) - e_I(t) &= T_c^+(z)[y(t) - e_0(t)] + T_u^+(z)e_u^+(t), \end{aligned} \tag{2b}$$

where the innovation processes do not contain the measurement noises and the transfer functions differ from that of the noisy system. Identifiability of EIV models are discussed in Refs [6, 7, 12].

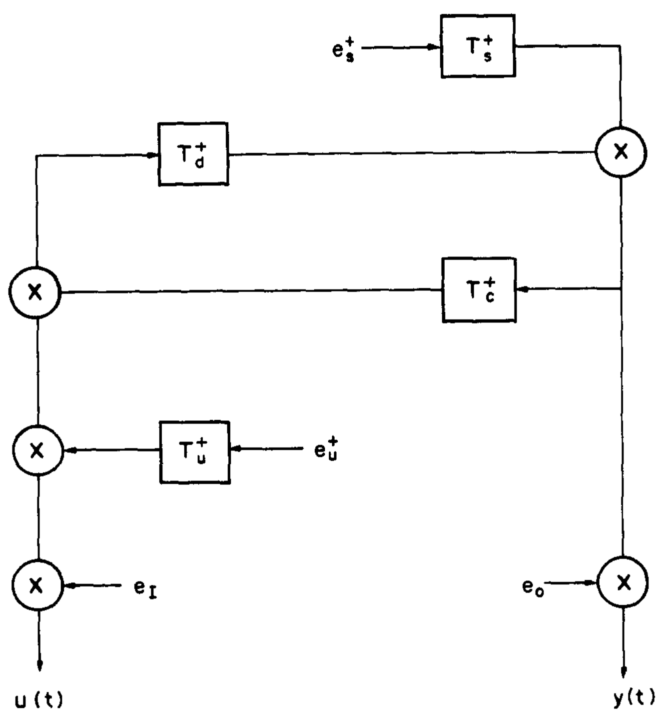

Fig. 1

2. IDENTIFICATION OF EIV SYSTEMS OPERATING IN OPEN LOOP

In the first step of the algorithm the noisy model of the $u(t) = T_u(z)e_u(t)$ is presented. The $e_u(t)$ innovation process and one step ahead prediction error contains the $e_l(t)$ white measurement noise of given covariance matrix $Ee_le_l^T$ and

$$T_u(z) = I + T_{u1}z^{-1} + T_{u2}z^{-2} + \cdots. \tag{3}$$

The rational innovation representation of the input process is to be determined in the form of

$$\begin{aligned} x(t+1) &= F_u x(t) + K_u e_u(t), \\ u(t) &= H_u x(t) + e_u(t). \end{aligned} \tag{4}$$

The $T_{ui}, i \geqslant 1$, Markov parameters have the forms of $T_{ui} = H_u F_u^{i-1} K_u$ and

$$T_u(z) = H_u(zI - F_u)^{-1}K_u + I. \tag{5}$$

It has been assumed that the dimension of the state vector is finite. The stochastic realization problem can be posed in the following way: given the covariance function

$$\Lambda_u(i) = Eu(t)u^{\mathrm{T}}(t-i), \quad i = 0, 1, 2, \ldots \tag{6}$$

of a zero mean stationary fully rank Gaussian process $u(t)$, find a representation of type (4), i.e. the H_u, F_u, K_u quantities and the dimension of the state vector are to be determined. The essence of the algorithm is the orthogonal projection of the future of the input signal on its present and past which results the spectral factorization of the density function $S_u = \Sigma_{-\infty}^{\infty} \Lambda_u(i)z^{-i}$ in the form of $T_u(z)Ee_ue_u^T T_u^*(z)$, where the asterisk denotes conjugate transpose.

Approximating the model order $n = \dim x_u = \operatorname{rank} H_u$ based on the $\delta_i, i = 1, 2, \ldots, n$, canonical correlations, δ_i are computed from Ref. [2]

$$|H_u R^{-1} H_u^{\mathrm{T}} R^{-1} - \delta^2 I| = 0, \tag{7}$$

where $|\cdot|$ denotes determinant and

$$H_u = \begin{bmatrix} \Lambda_u(1) & \Lambda_u(2) & \cdot \\ \Lambda_u(2) & \Lambda_u(3) & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \quad \text{and } R = \begin{bmatrix} \Lambda_u(0) & \Lambda_u^{\mathrm{T}}(1) & \cdot \\ \Lambda_u(1) & \Lambda(0) & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}. \tag{8}$$

The mutual information between the future and the past of the input process is

$$I_n = -\frac{1}{2}\sum_i \log(1 - \delta_i^2), \quad i = 1, 2, \ldots, n, \tag{9}$$

and an $\hat{n} \leqslant n$ can be accepted when $I_{\hat{n}}$ is reasonably close to I_n.

In the next lines we should like to show the computations of the H_u, F_u, K_u parameters with Faurre's algorithm. With the Ho–Kalman algorithm [4] on the H_u Hankel matrix we get the factorization of the covariance function $\Lambda_u(i)$ in the form of $\Lambda_u(i) = H_u F_u^{i-1} G_u, i = 1, 2, \ldots,$ and according to Faurre's algorithm [1]: let us solve iteratively

$$\begin{aligned} Exx^{\mathrm{T}} &= F_u Exx^{\mathrm{T}} F_u^{\mathrm{T}} + K_u Ee_u e_u^{\mathrm{T}} K_u^{\mathrm{T}}, \\ G_u &= F_u Exx^{\mathrm{T}} H_u^{\mathrm{T}} + K_u Ee_u e_u^{\mathrm{T}}, \\ Euu^{\mathrm{T}} &= H_u Exx^{\mathrm{T}} H_u^{\mathrm{T}} + Ee_u e_u^{\mathrm{T}}, \end{aligned} \tag{10}$$

which is the quadratic form of equation (4). The solutions are denoted by Exx^{T}, $Ee_ue_u^{\mathrm{T}}$ and K_u as well. There exists a canonical form called balanced realization when $Exx^{\mathrm{T}} = \langle \delta_i \rangle$ [5].

It is possible to show that the first block column of the $H_u R^{-1}$ matrix gives the $T_{ui}, i = 1, 2, \ldots,$ Markov parameters [2] and composing a Hankel matrix from T_{ui} the H_u, F_u and K_u parameters are computable with the Ho–Kalman algorithm in a simple way.

When the EIV model is computed the main point is that Euu^{T} is replaced with $Euu^{\mathrm{T}} - Ee_l e_l^{\mathrm{T}}$ and $Ee_u E_u^{\mathrm{T}}$ with $Ee_u e_u^{\mathrm{T}} - Ee_l e_l^{\mathrm{T}}$ in the algorithm thus $T_u^+(z)$ and $Ee_u^+ e_u^{+\mathrm{T}}$ are identifiable.

The second step of the algorithm is the computation of the $T_d^+(z)$ transfer function in the form of $T_d^+(z) = H_d^+(zI - F_d^+)^{-1}K_d^+ + K_{d0}^+$. In the white noise input case the determination of the $T_d(z)$ transfer function was discussed by the authors [13] applying a realization algorithm. The essence of the algorithm is the orthogonal projection of the future and present of the output process onto the present and past of the input process. In the case of open loop control systems the projection happens only on the past of the input process since these systems have delay in the forward path. Because of the input and output measurement noises are not correlated for all t the $T_d^+(z)$ transfer function is identifiable in the EIV model case as well.

If $\Lambda_{yu}(i) = Ey(t)u^{\mathrm{T}}(t-i), i = 0, 1, 2, \ldots$ denote the crosscovariances and

$$H_d = \begin{bmatrix} \Lambda_{yu}(0) & \Lambda_{yu}(1) & \cdot \\ \Lambda_{yu}(1) & \Lambda_{yu}(2) & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}, \tag{11}$$

then the $H_d R^{-1}$ matrix represents the projection and it should be factored with the Ho–Kalman algorithm but it is not a Hankel matrix. In the R matrix Euu^{T} is modified for $Euu^{\mathrm{T}} - Ee_l e_l^{\mathrm{T}}$. Recognize that the first block row of the $H_d R^{-1}$ matrix gives the T_{di} parameters and composing a Hankel matrix based on these parameters and factoring it with the Ho–Kalman algorithm we can compute the desired factorization of $T_d^+(z)$.

Another possibility is the orthogonal projection of the future and present of the output process onto the whitened present and past of the input process and this projection results a T_{dw}^+ transfer function with which $T_d^+(z) = T_{dw}^+(z)[T_u^+(z)]^{-1}$ is computable. The first block column of the $H_u R^{-1}$ matrix gives the Markov parameters of T_{dw}^+ since $T_u^+(\infty) = I$. This result may seem unexpected as each of the T_d^+, T_u^+ and T_{dw}^+ transfer functions can be identified based on the $H_u R^{-1}$ matrix

$$H_d R^{-1} = \begin{bmatrix} T_{d0}^+ & T_{d1}^+ & T_{d2}^+ & \cdots \\ T_{dw1}^+ & & & \\ T_{dw2}^+ & & & \\ \vdots & & & \end{bmatrix}, \tag{12}$$

when T_d^+ is a minimum phase, $T_{d0}^+ = T_{dw0}^+$.

The third step of the algorithm the computation of the factorization of the $T_s^+(z)$ in the EIV model. This task will be fulfilled based on the spectral density functions of the input and the output. When $\Lambda_y(i) = Ey(t)y^{\mathrm{T}}(t-i), i = 0, 1, 2, \ldots,$ denote the covariance function of the output process and

$$S_y = \sum_{-\infty}^{\infty} \Lambda_y(i)z^{-i},$$

$$S_u = \sum_{-\infty}^{\infty} \Lambda_u(i)z^{-i} = T_u^+ Ee_u^+ e_u^{+\mathrm{T}} T_u^{+*} + Ee_l e_l^{\mathrm{T}}, \tag{13}$$

than

$$\begin{aligned} T_s^+ Ee_s^+ e_s^{+\mathrm{T}} T_s^* &= Sy - Ee_0^+ e_0^{+\mathrm{T}} - T_d^+ T_u^+ Ee_u^+ e_u^{+\mathrm{T}} T_u^{+*} T_d^{+*} \\ &= Sy - Ee_0^+ + e_0^{+\mathrm{T}} - T_{dw}^+ Ee_u^+ e_u^+ T_{dw}^{+*}, \end{aligned} \tag{14}$$

can be computed and its spectral factorization with Faurre type realization algorithms gives $Ee_s^+ e_s^{+\mathrm{T}}$ and

$$T_s^+(z) = H_s(zI - F_s^+)^{-1}K_s^+ + I, \tag{15}$$

which is minimal phase transfer function.

3. IDENTIFICATION OF EIV MODELS OPERATING IN CLOSED LOOP

Applying Faurre type realization for the factorization of the rational positive definite spectral density function

$$S = \begin{bmatrix} S_y - Ee_0e_0^{\mathrm{T}} & S_{yu} \\ S_{uy} & S_u - Ee_le_l^{\mathrm{T}} \end{bmatrix}, \tag{16}$$

of the stationary joint $\begin{bmatrix} y(t) \\ u(t) \end{bmatrix}$ zero mean Gaussian process one can get a noncausal model in the form of

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} f_s(t) \\ f_u(t) \end{bmatrix} = Wf(t), \tag{17}$$

where $W(z)$ is a minimum phase transfer function, i.e. it is stable and inverse stable. For noncausal EIV models see Picci–Pinzoni [6] and Deistler [7]. Note that there exists canonical form when the covariance matrix of the state vector is diagonal and contains the canonical correlations but it is not a balanced realization.

Let us consider the identifiability of a causal EIV model with Faurre type realization algorithms when it would be necessary based on equation (2b)

$$\begin{aligned} y(t) - e_0(t) &= [I - T_d^+ T_c^+]^{-1}[T_d^+ T_c^+ e_u^+(t) + T_s^+ e_s^+(t)] \triangleq w_{11}^+ e_s^+(t) + w_{12}^+ e_u^+(t) \\ u(t) - e_l(t) &= [I - T_c^+ T_d^+]^{-1}[T_c^+ T_s^+ e_s^+(t) + T_u^+ e_u^+(t)] \triangleq w_{21}^+ e_s^+(t) + w_{22}^+ e_u^+(t), \end{aligned} \tag{18}$$

and the transfer functions are computable in the forms of

$$\begin{aligned} T_d^+(z) &= w_{12}^+ w_{22}^{+\,-1}, \\ T_s^+(z) &= w_{11}^+ - w_{12}^+ w_{22}^{+\,-1} w_{21}^+, \\ T_c^+(z) &= w_{21}^+ w_{11}^{+\,-1}, \\ T_u^+(z) &= w_{22}^+ w_{11}^{+\,-1} w_{12}^+. \end{aligned} \tag{19}$$

The identifiability conditions of a causal model can not be ensured with Faurre type realization algorithms—i.e. there is a delay in the forward path and $e_s^+(t)$ is orthogonal to $e_u^+(t)$—since w_{ij}, $i = 1, 2$, $j = 1, 2$ and $f_s(t)$ and $f_u(t)$ differ from the proper quantities of equation (18) and $f_s(t)$ is not orthogonal to $f_u(t)$. Otherwise $W(z)$ is a minimal phase transfer function and this fact is not assumed at the causal EIV model.

Applying succesive Gram–Schmidt orthogonalization of the S spectral density function the causal EIV model is identifiable see Refs [9, 10].

In the next lines we give an investigation which shows the nature of the identifiability problem with Faurre type realization algorithms. From equation (2b) one can get the noise models

$$\begin{aligned} T_s^+ Ee_s^+ e_s^{+\mathrm{T}} T_s^* &= [I - T_d^+ T_c^+][S_y - Ee_0e_0^{\mathrm{T}}][I - T_d^+ T_c^+]^* - T_d^+ [S_u - Ee_le_l]T_d^* \\ &\quad - [I - T_d^+ T_c^+]S_{yu} T_d^{+*} - T_d^+ S_{uy}[I - T_d^+ T_c^+]^*, \\ T_u^+ Ee_u^+ e_u^{+\mathrm{T}} T_u^* &= [I - T_c^+ T_d^+][S_u - Ee_l^+ e_l^{+\mathrm{T}}][I - T_c^+ T_d^+]^* - T_c^+ [S_y - Ee_0e_0^{\mathrm{T}}]T_c^* \\ &\quad - [I - T_c^+ T_d^+]S_{uy} T_c^* - T_c^+ S_{yu}[I - T_c T_d]^*. \end{aligned} \tag{20}$$

These equations show that a causal EIV model is identifiable if $T_d^+(z)$ and $T_c^+(z)$ are identifiable. Note that $T_c(z)$ is frequently known at industrial systems except in the case when a manual operator is acting in the backward path.

The identification of these transfer functions with Faurre type realization algorithms can not be fulfilled as in the open loop case because of the presence of the feedback path.

4. CONCLUSIONS

Applying Faurre type realization algorithms for the spectral factorization of stationary discrete time series coming from open or closed loop systems causal EIV or noisy models are identifiable in the open loop case and noncausal models are identifiable in the closed loop case.

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